

Computer-aided design of slider bearings in magnetic disk files

by K. L. Deckert

This paper reviews the application of lubrication theory to slider bearings in magnetic disk files. For more than thirty years, slider bearings have been used to maintain close and precise spacings between recording transducer and recording medium in disk files. Computer modeling has been central to the design and performance analysis of these systems. The topics covered are the basic design, sensitivity and tolerance analysis, dynamic characteristics, and response to disk excitations from the disk. The main purpose of this paper is to review the use of computer modeling in design of slider bearings; however, the discussion of slider modes in the slider dynamics section is new.

Introduction

More than one hundred years have passed since the publication of Osborne Reynolds' seminal paper on the theory of hydrodynamic lubrication [1]. Since then, the

application of hydrodynamic lubrication theory has played a major role in the development of the mechanical systems that are so important for the functioning of modern society. This paper is about an application of hydrodynamic lubrication theory to a critical part of the magnetic disk storage devices used by most of today's computers.

A fixed-disk drive has one or more metal disks stacked on a rotating spindle. The disk surfaces are coated with a thin magnetic material on which data can be recorded and read. A transducer, or read/write head, records or reads the data from narrow concentric tracks of the magnetic material. The transducer, in turn, is attached to a "slider" suspended close to the recording surface by a mechanism that can find and follow the recorded data tracks. The spacing between slider and disk is controlled very precisely by a lubricating film of air that forms between them. The combination of slider, air film, and moving disk surface is an air-lubricated slider bearing and is commonly called a slider bearing or an air bearing.

It is possible to design the slider so that it operates reliably with a spacing of a fraction of a micrometer. In general, higher data recording densities require smaller head-to-disk spacings; at the same time, smaller spacings impose tighter tolerances on the smoothness of the disk surface in order to avoid failure due to excessive contact between the slider and disk. The problem for designers of slider bearings in disk files is to satisfy the competing

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requirements of high-density recording and reliability of the slider-disk interface.

The magnetic disk storage drive was first developed by IBM and introduced as the IBM 305 RAMAC system in 1957 [2]. This first system used a hydrostatic bearing (pressurized by an external air source), but succeeding drives soon switched to hydrodynamic bearings (pressurized by the boundary layer of air moving with the rotating disk surface). The most common air bearing in use today is based on the so-called taper-flat bearing, in which the air bearing surfaces consist of two or more narrow rails resembling skis, with short leading taper sections followed by longer, flat portions. This is the type of bearing that first appeared in 1973 in the IBM 3340 disk file, which was developed with the internal code name "Winchester." **Figure 1** is a photograph of the Winchester slider and suspension.

When the first disk files were being developed, it was recognized that computer modeling was needed to help in the design of sliders, and pioneering work was done by Gross [3] and Michaels [4] at IBM. The computer program they created has been continually modified and improved. It was, and still is, a very important tool for slider bearing design. Details of the evolution and innovations in slider design can be found in the 25th anniversary issue of this journal [5].

Theoretical considerations

Figure 2 is a sketch of a two-rail, taper-flat slider, showing the coordinate system and slider position variables. The flying height h_0 is the distance between the read/write element and the disk surface. The spacing function $h(x, y, t)$ between disk and air bearing surface depends upon the geometry of the slider, the flying height, and the slider pitch and roll angles θ and ϕ . The air pressure between slider and disk is governed by the time-dependent compressible Reynolds equation

$$12\mu \frac{\partial}{\partial t}(ph) = \nabla \cdot [h^3 p \nabla p + 6\lambda_a p_a h^2 \nabla p - 6\mu \tilde{V} ph], \quad (1)$$

where

$p(x, y, t)$ = bearing pressure,
 $h(x, y, t)$ = bearing spacing,
 $\tilde{V}(x, y)$ = disk surface velocity vector,
 μ = viscosity of air,
 λ_a = mean free path of air molecules,
 p_a = ambient air pressure.

For most problems, the slider, constrained by the suspension, has but three degrees of freedom of motion given by the z motion of the pivot point together with the pitch and roll motions, θ and ϕ . The equations of motion of the slider are then given by

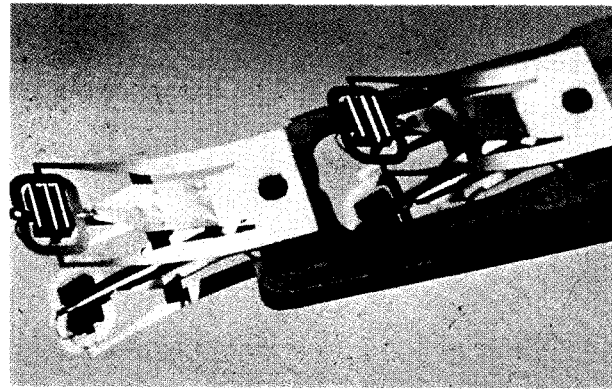


Figure 1

Winchester slider and suspension.

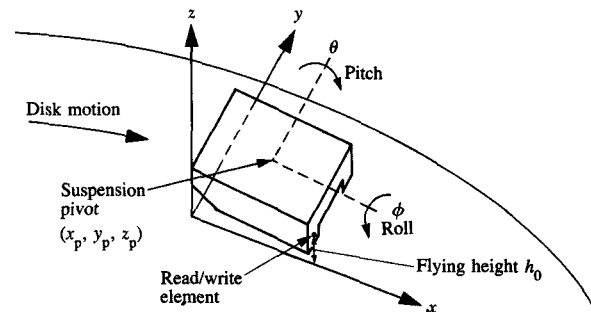


Figure 2

Slider and disk showing the coordinate system.

$$\begin{aligned} m\ddot{z} + k_z z &= L_s + \int_B (p - p_a) dx dy, \\ I_\theta \ddot{\theta} + k_\theta \theta &= P_s + \int_B (p - p_a)(x_p - x) dx dy + S_x z_p, \\ I_\phi \ddot{\phi} + k_\phi \phi &= R_s + \int_B (p - p_a)(y - y_p) dx dy + S_y z_p, \\ \dot{S} &= \int_B \left[-\frac{h}{2} \nabla p + \frac{\mu \tilde{V}}{h} \left(\frac{ph}{ph + \lambda_a p_a} \right) \right] dx dy, \end{aligned} \quad (2)$$

where

B = air bearing surface,
 (m, I_θ, I_ϕ) = slider mass and pitch and roll inertias,
 (k_z, k_θ, k_ϕ) = suspension spring stiffnesses,

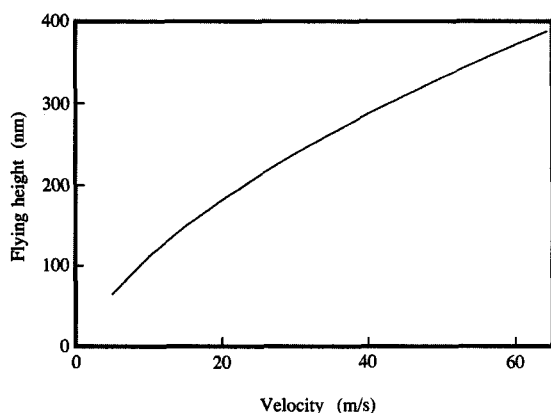


Figure 3

Velocity/flying height curve for 3380-type slider.

(L_s, P_s, R_s) = suspension load and pitch and roll moments,

(x_p, y_p, z_p) = suspension pivot coordinates,

$(S_x, S_y) = \vec{S}$ = air drag on the air bearing surface.

Equation (1) contains the term $6\lambda_a p_a h^2 \nabla p$, which is not in the classical Reynolds equation. This term appears when the lubricating fluid, in this case, air, "slips" relative to the air bearing surface instead of sticking to the surface, as in the classical case. It was first derived by Burgdorfer [6], who assumed that molecules of air are diffusely reflected from the surfaces. In more detailed analysis, molecular reflection depends on the type of surface [7]. A fundamental assumption in the derivation of the Reynolds equation is that the mean free path, λ , of a molecule of the lubricating fluid is small compared with the spacing between the bearing surfaces. As the bearing spacing is reduced to meet ever-higher recording density requirements, this assumption is violated, and the validity of the Reynolds equation becomes suspect. The equation with slip was designed to improve the validity at smaller spacings, and experience shows that the results it gives are better than expected [8]. More accurate solutions at very small spacings involve the solution of the Boltzmann equation [9].

Since these equations have no general analytic solutions, they must be solved numerically. The numerical solution of Equation (1) has a long history of its own, which is not covered in detail here. Successful programs have been written to compute solutions based on finite differences [10] and finite elements [11, 12]. At

IBM, the main slider design program that has evolved through the years solves the steady-state form of the equations, obtained by setting time-dependent terms to zero. This finite difference program, subsequently referred to as the Steady State Air Bearing Program, is based on a technique discussed in the review paper of Castelli and Pirvics [13]. A separate program for solving the time-dependent equations, subsequently referred to as the Dynamic Air Bearing Program, is used to study slider dynamics.

In what follows, four of the many possible uses of these programs in slider design are discussed. These are the design of the air bearing geometry, sensitivity and tolerance analysis, slider air bearing resonant frequencies, and the response of the system to disk runout. The ultimate goal of the air bearing designer is to produce a design that meets magnetic recording requirements with maximum reliability of operation and minimum cost of production.

Air bearing geometry

One important function of the slider in a disk file is to support the magnetic read/write element at a prescribed flying height from the disk surface, which is determined by the magnetic properties of the recording surface, the desired recording density, and the minimum allowable error rate. It should be noted that errors in recording are corrected by redundancy coding, so that the probability of final error is extremely small. In addition to flying height, the designer may have requirements for the slider flying pitch and roll angles. The combination of steady-state flying height and pitch and roll angles (h_0, θ, ϕ) is called the flying attitude of the slider.

The Steady State Air Bearing Program is most useful for finding a desirable air bearing geometry. Most often, the designer works within a set of constraints, such as overall slider dimensions, placement of the read/write element, and shapes preferred for manufacturability. The design process is usually evolutionary, in that a previous design is modified to account for changes in disk rotational speed, suspension characteristics, or new flying-attitude requirements.

The most general calculation performed by the steady-state program is finding the flying-attitude values so that the resulting hydrodynamic pressure, integrated over the air bearing surface, gives a lifting force and pitch and roll moments that balance the external forces and moments from the suspension. The result is called a full steady-state flying-attitude solution. In some cases, all that the user requires are the force and moments for a given set of flying-attitude values. These are called fixed-attitude solutions. In addition, the designer can choose to calculate a partial steady-state flying-attitude solution in which one or more of the flying-attitude values are held

fixed. A good example is the case in which the roll angle is held fixed at zero and the disk velocity is assumed constant over the width of the slider. Then, for symmetric air bearing geometries, the pressure has symmetries that can be exploited to significantly reduce the required computation. The usefulness and value of the steady-state program as a design tool is greatly enhanced by the availability of a wide variety of such options.

Once a design is found that produces the desired flying attitude, the steady-state program can be used to calculate useful performance characteristics. Some possibilities include calculation of flying attitude versus such variables as disk surface velocity, suspension load force, or taper angle. **Figure 3** is an example of a curve of flying height versus velocity for the slider used in the IBM 3380 disk drive. This curve can be used to determine the so-called take-off/landing velocity of the disk, which affects the time that the slider is in contact with the disk during start/stop.

Static air bearing stiffnesses can be found by calculating the change in lift force and moments of the bearing from fixed-attitude solutions that are close to the steady-state solution.

Sensitivity and tolerance analysis

There are a surprising number of physical parameters that influence the flying attitude of a slider. During the fabrication of the slider, these parameters will vary statistically from their design values; thus, the flying attitude of a sample of those sliders will also have a statistical distribution. The slider designer must understand the parameters that influence the slider flying attitude. He needs to know their sensitivities and their expected statistical distribution in the fabrication process. From these data he can determine the relative importance of each parameter and make more intelligent decisions in specifying their design values.

Let $\bar{x}$ be a vector whose components are the important slider parameters, and let the dependence of the flying height h on the parameters be $h = f(\bar{x})$. Then the first-order approximation to the change in flying height Δh due to small changes in the parameters Δx_i is given by

$$\Delta h = \frac{\partial f}{\partial x_1} \Delta x_1 + \frac{\partial f}{\partial x_2} \Delta x_2 + \cdots + \frac{\partial f}{\partial x_n} \Delta x_n. \quad (3)$$

The partial derivatives in the above equation are called sensitivity coefficients. The Steady State Air Bearing Program can be used to estimate the sensitivity coefficients by computing the change in flying height due to a small change in each parameter individually and then forming the corresponding difference quotients. Sensitivity coefficients for the pitch and roll flying attitudes can be computed at the same time.

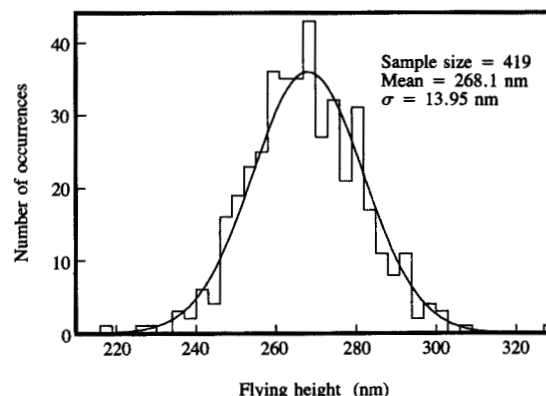


Figure 4

Flying height distribution with normal fit.

The sensitivity coefficients are used in tolerance analysis, which is the determination of how the fabrication tolerances of individual slider parameters affect the final flying-attitude tolerances. The allowable tolerance of the flying attitude of the sliders that are built into the disk file is determined by performance and reliability considerations. Once these are given, the slider designer and process engineers determine the tolerances of each individual parameter so as to maximize the quality of the finished product while minimizing the fabrication cost.

Experience shows that the statistical variation of each process parameter can be approximated by a normal distribution. If σ_i is the standard deviation of one of these parameters, then the flying-height deviation due to this parameter alone would be

$$\sigma_h = \frac{\partial f}{\partial x_i} \sigma_i. \quad (4)$$

When normal distributions are combined, the variances or squares of the standard deviations add linearly, so an equation for the total flying-height variance resulting from individual parameter variances can be written

$$\sigma_h^2 = \left(\frac{\partial f}{\partial x_1} \sigma_1 \right)^2 + \left(\frac{\partial f}{\partial x_2} \sigma_2 \right)^2 + \cdots + \left(\frac{\partial f}{\partial x_n} \sigma_n \right)^2. \quad (5)$$

Details can be found in a paper [14] that reports on results using an experimental slider at the IBM GPD Development Laboratory in San Jose. **Figure 4** shows the histogram of flying-height measurements of a sample of the sliders studied in that paper, together with a fitted normal distribution. The value of flying-height standard deviation as calculated in [14], using Equation (5), is

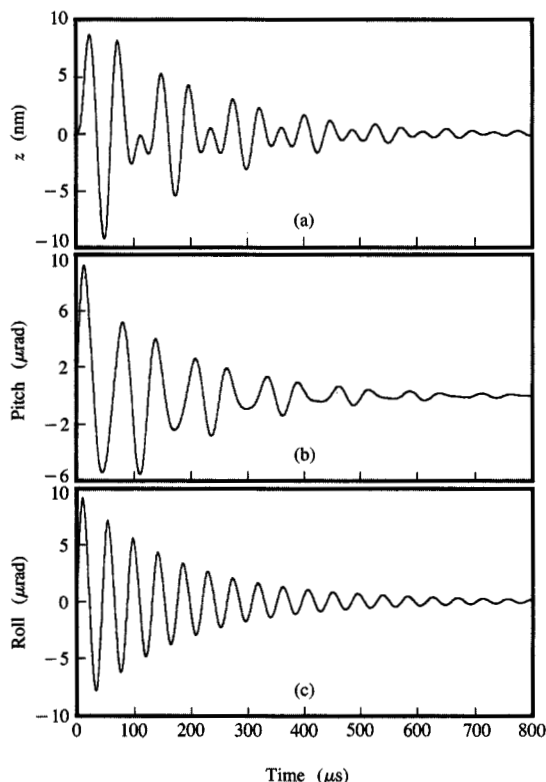


Figure 5

(a) Pivot, (b) pitch, and (c) roll impulse response of 3380-type slider.

14.34 nanometers. This compares very well with the measured value of 13.95 nanometers.

Slider dynamics

The Dynamic Air Bearing Program solves Equations (1) and (2) for a variety of forcing functions and conditions not included in these equations. For a force applied at the suspension pivot equal to that applied by the actuator when moving the slider from one data track to another, one can study the resulting flying-height variations and dynamics. If a given non-flat disk surface function $s(x, y, t)$ is included in the calculation of the spacing function $h(x, y, t)$, one can study the effect of the disk surface on the slider dynamics. These are forced motions of the system, which can be better understood if the free vibrations or resonant modes of the system are known.

The resonant modes of the slider air bearing system can be found by analysis of the system impulse response. The Dynamic Air Bearing Program produces impulse responses by solving the equations starting from steady

state with initial velocities given to the flying-attitude parameters. Mathematically, the resulting solution is identical to the solution using zero initial velocities and an impulsive forcing function. **Figure 5** shows the z motion at the pivot, the pitch motion, and the roll motion due to simultaneous pitch and roll impulses for a 3380-type slider operating at 40 meters per second. Because the system is nonlinear, the strength of the impulse used must be less than unity in order to keep the amplitude of the resulting motion small. It can be seen that the pivot and pitch motions contain two or more frequencies, while the roll motion contains essentially one frequency. This is characteristic of symmetric sliders operating on large disks. Asymmetries, such as rails of different widths or the suspension pivot point offset from center, will increase the coupling of the roll motion with pitch and spacing motions.

For small-amplitude oscillations, one can model this system as a linear, three-degree-of-freedom system of masses, springs, and dampers. The general impulse response of such a system can be written as

$$\begin{aligned} z &= \sum_{j=1}^3 a_{1j} e^{-\zeta_j \omega_j t} \sin \sqrt{1 - \zeta_j^2} \omega_j t, \\ \theta &= \sum_{j=1}^3 a_{2j} e^{-\zeta_j \omega_j t} \sin \sqrt{1 - \zeta_j^2} \omega_j t, \\ \phi &= \sum_{j=1}^3 a_{3j} e^{-\zeta_j \omega_j t} \sin \sqrt{1 - \zeta_j^2} \omega_j t, \end{aligned} \quad (6)$$

where

a_{ij} = amplitude coefficient,
 ω_j = resonant frequency,
 ζ_j = damping factor.

The amplitudes, frequencies, and damping coefficients that give a best fit to the impulse responses from the Dynamic Air Bearing Program can be found by the use of a nonlinear least-squares fitting technique [15]. When this is applied to the data shown in Figure 5, the following frequencies and damping factors are obtained (note that the frequencies are given in hertz rather than radians per second):

$$\begin{aligned} f_1 &= 25\,250, & f_2 &= 16\,800, & f_3 &= 24\,350, \\ \zeta_1 &= 0.033, & \zeta_2 &= 0.055, & \zeta_3 &= 0.016. \end{aligned} \quad (7)$$

One can gain insight into the normal modes of the slider by solving Equation (6) for the damped sinusoids in terms of z , θ , and ϕ . First define the dimensionless vector $\tilde{q}_j$, with three components

$$q_j = e^{-\zeta_j \omega_j t} \sin \sqrt{1 - \zeta_j^2} \omega_j t \quad \text{for } j = 1, 2, 3. \quad (8)$$

Thus,

$$\begin{aligned}
z &= \sum_{j=1}^3 a_{1j} q_j, \\
\theta &= \sum_{j=1}^3 a_{2j} q_j, \\
\phi &= \sum_{j=1}^3 a_{3j} q_j.
\end{aligned} \tag{9}$$

Then rewrite Equations (6) in terms of the vector $\tilde{q}$ and solve the equations for the vector $\tilde{q}$ in terms of z , θ , and ϕ . When the amplitude coefficients a_{ij} from the best fit of the data shown in Figure 5 are substituted, the result is given by

$$\tilde{q} = \begin{bmatrix} -3.48 \times 10^5 (z - 0.081\theta - 0.351\phi) \\ 4.74 \times 10^5 (z + 0.215\theta + 0.003\phi) \\ -3.37 \times 10^6 (z - 0.083\theta + 0.040\phi) \end{bmatrix} \tag{10}$$

To understand these equations, one should consider the expression for the displacement from steady state $d(x, y)$ of a given point on the slider as a function of the slider attitude. The angles involved are of the order of microradians, so that the tangent of an angle can be approximated by the angle; therefore,

$$d(x, y) = z - (x - x_p)\theta + (y - y_p)\phi. \tag{11}$$

Note that the negative sign for the θ term results from the choice of coordinates and slider orientation in Figure 2. Comparing (10) with (11) reveals that the modes q_j for $j = 1, 2, 3$ can be thought of as motions of distinct points on the slider. For example, the first mode, q_1 , is the motion of a point whose x, y coordinates are given by

$$\begin{aligned}
-0.081 &= -(x - x_p), \\
-0.351 &= (y - y_p).
\end{aligned} \tag{12}$$

The length unit of the Dynamic Air Bearing Program is centimeters, so q_1 represents a point located 0.081 cm in x and -0.351 cm in y from the pivot point. Mode points associated with the other two components of $\tilde{q}$ are similarly defined. Suppose the initial conditions were chosen so that only one of the modes was activated, with the amplitude of the other two modes being zero. Then the points on the slider associated with the inactive modes would have zero displacement, and, since the slider is a rigid body, its motion would be a rotation about an axis passing through these inactive-mode points.

Figure 6 is a sketch of the slider showing the three mode points and the resulting axes of rotation for each mode. It can be seen that mode 1 is a roll mode, mode 2 is a pitch mode, and mode 3 is a combined pitch and roll mode. In general, the force-free motion of the slider is a

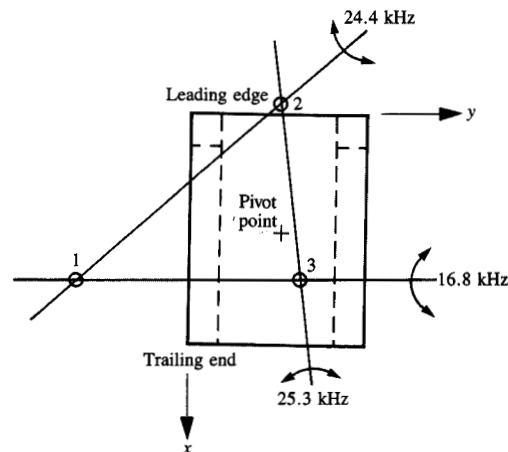


Figure 6

Slider showing modal axes of rotation.

linear combination of these three modes. This analysis gives significant insight for understanding experimental measurements.

Response to disk runout

One purpose of the air bearing is to keep the slider from contacting the disk during operation. Contact does occur during the starting and stopping phases when the slider "takes off" from and "lands" on the disk surface. An interesting simulation of start/stop operation can be found in Benson and Talke [16]. The subject of the following is the effect of disk motion on the performance of the bearing.

The steady-state program assumes an ideal, perfectly flat disk; however, real disks are not perfect, and the disk surface as seen by the slider will move in the z direction depending on the disk surface geometry and the method of attaching the disk to the spindle. This motion is called runout, and its magnitude is of the order of micrometers. Since the dimensions of the slider are of the order of millimeters, the disk surface in the air bearing can be represented by a second-order approximation as follows:

$$S(x, y) = c + \frac{\partial S}{\partial x}x + \frac{\partial S}{\partial y}y + \frac{\partial^2 S}{\partial x^2}x^2 + \frac{\partial^2 S}{\partial x \partial y}xy + \frac{\partial^2 S}{\partial y^2}y^2. \tag{13}$$

Each term in this expansion can be associated with a corresponding term of $h(x, y, t)$ in Equation (1). The constant term c is an overall change of flying height; the

first partial derivatives are slopes in the x and y directions, thus equivalent to slider pitch and roll; and the second partial derivatives are related to curvature. The second partial derivative with respect to x multiplied by x^2 is commonly called the crown, and the second partial derivative with respect to y is known as camber or cross-curvature. The remaining mixed partial derivative term represents the twist of the surface. Therefore, the effect of a moving disk surface can be modeled by adding time-dependent spacing, pitch, roll, crown, camber, and twist terms to $h(x, y, t)$. The time-dependency can be represented as sines and cosines, with amplitude, frequency, and phase determined by the type of disk runout (e.g., warped disks or imperfect spindle bearings).

The disk runout observed experimentally occurs at frequencies well below the air bearing resonant frequencies. At low frequencies, the stiffness of the air bearing causes the slider to follow the spacing, pitch, and roll of the disk surface except for a small component due to the inertia of the slider. To estimate the magnitude of the inertial component, recall that in a simple harmonic oscillator the amplitude of forced oscillations at a frequency much lower than the resonant frequency ω_0 is given by $F/(m\omega_0^2)$, where F is the magnitude of the applied force. Disk runout is not a force, but since F/m is an acceleration, the amplitude of the spacing modulation can be estimated by $\ddot{z}/\omega_0^2$, where $\ddot{z}$ is the disk surface vertical acceleration. For a sinusoidal surface motion of amplitude A and frequency ω , the amplitude of the acceleration is $A\omega^2$. Thus, the amplitude of response is $A(\omega/\omega_0)^2$. Experimental data reported by Zhu and Bogy [17] show that this is but a small part of the observed spacing modulation; consequently, the most important time-dependent disk surface features are related to second-degree surface curvature terms. As a result, slider modulation due to low-frequency disk runout can be modeled by measuring the effective disk curvature across the air bearing and multiplying this by the corresponding air bearing curvature sensitivity coefficient.

Concluding remarks

For more than thirty years, computer modeling has been of primary importance in the design of slider bearings in magnetic disk files. Flying attitude, tolerances, and dynamic performance of slider bearings in magnetic disk files can be calculated by computer programs based on solving the Reynolds equation. The results of such calculations are used to specify the design parameters, and predict the performance of slider bearings.

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